

Negative magnetoresistance of ultra-narrow superconducting nanowires in the resistive state

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We present a phenomenological model qualitatively explaining negative magnetoresistance in quasi-one-dimensional superconducting channels in the resistive state. The model is based on the assumption that fluctuations of the order parameter (phase slips) are responsible for the finite effective resistance of a narrow superconducting wire sufficiently close to the critical temperature. Each fluctuation is accompanied by an instant formation of a quasi-normal region of the order of the non-equilibrium quasiparticle relaxation length 'pinned' to the core of the phase slip. The effective time-averaged voltage measured in experiment is a sum of two terms. First one is the conventional contribution linked to the rate of the fluctuations via the Josephson relation. Second term is the Ohmic contribution of this quasi-normal region. Depending on material properties of the wire, there might be a range of magnetic fields where the first term is not much affected, while the second term is effectively suppressed contributing to the experimentally observed negative magnetoresistance.

I. INTRODUCTION

It is a text-book knowledge that shape of a superconducting phase transition $R(T)$ of a homogeneous sample is determined by fluctuations of the order parameter. The rounded top part (the onset of superconductivity) is accounted for nucleation of the superconducting phase shunting the normal current. The effect can be observed in superconducting systems irrespectively of their dimensionality, set by the temperature-dependent coherence length $\xi(T)$. At the bottom (low temperature) part of the transition, dimensionality plays an essential role. As soon as at least one channel of supercurrent nucleates, in 2D and 3D systems spots of normal phase do not contribute to zero DC resistance, and the $R(T)$ dependence has an abrupt bottom part. Situation is qualitatively different in long quasi-1D systems with cross-section $\sigma \leq \xi^2$ where there exist only one parallel channel of a supercurrent. Fluctuations of the order parameter destroy phase coherence in the wire and, hence, finite resistance is produced. In experiment this phenomenon manifests itself as a rounded low temperature part of the $R(T)$ phase transition. In a very simplified way such a behavior can be understood as for sufficiently long 1D channel of length $L \gg \xi$ there is always a finite probability of a fluctuation to drive instantly a fraction of the wire into a normal state. As there is only one supercurrent channel, these events cause momentary energy dissipation. The instant voltage jumps, being integrated in time, are commonly interpreted as a finite DC resistance. If at a given measuring current I the rate of fluctuations is Γ_{PS} , then the effective resistance is given by the Josephson relation: $R_{PS} \equiv \langle V \rangle_{PS}/I = h\Gamma_{PS}/2eI$. Two different scenarios have been developed defining the rate of fluctuations of the order parameter (phase slips) in 1D superconducting channels: thermal (TAPS) [1] - [3] and quantum (QPS) [4] - [14]. The principal difference is that in TAPS the required energy for the fluctuation is provided by the 'classical' term $\sim k_B T$, while in QPS the relevant energy scale is $\sim h\Gamma_{QPS}$.

In early experiments on tin whiskers [15] it has been shown that the shape of the $R(T)$ transition can be nicely fitted by the TAPS model [1]. By modern standards the cross section of that samples $\sigma^{1/2} \sim 0.5 \text{ m}\mu$ is rather large, and the corresponding width of experimentally detectable $R(T)$ transition is about $\sim 1 \text{ mK}$. During the last decades the rapid progress in nanotechnology enabled study of much narrower superconducting channels reaching sub-10 nm scales [16] - [24]. In these ultra-narrow nanostructures the $R(T)$ transitions are much wider than the TAPS model predictions. The effect is associated with manifestation of the

QPS mechanism. Though the interpretation of results is still contradicting, the experiments have revealed several unusual features. Among them - 'enhancement' of superconductivity by magnetic field, which manifests itself either as negative magnetoresistance (nMR) or/and increase of the critical current I_c in magnetic field. The effect is observed in the very narrow quasi-1D superconducting channels in a resistive state below the critical temperature T_c [18], [24], [25]. A trivial explanation related to Kondo mechanism can be ruled out as concentration of magnetic impurities in these samples is negligible; the corresponding magnetic field is too small to polarize any imaginable magnetic momentum, and additionally the onset of superconductivity is not affected by a small magnetic field where nMR is observed. While various scenarios have been proposed [18], [6], [2], [26], [27] the matter is not yet settled and there is no commonly accepted explanations for the nMR phenomenon. In this paper we present a simple phenomenological model accounting the nMR effect to formation of a quasi-normal region 'pinned' to each fluctuation-mediated phase slip. The extension of this region is set by the quasiparticle relaxation length Λ_Q and is effectively suppressed by a weak magnetic field giving rise to the nMR.

II. THE MODEL

For concreteness we consider that the resistive state of a quasi-1D superconducting channel is govern solely by the TAPS mechanism. However, we believe that qualitatively the model should be applicable to QPS activation as well. It has been shown [1] that the rate of TAPS Γ_{PS} at a bias DC current I is:

$$\Gamma_{PS}(T, I) = \Omega(T, I, H) \exp \left[-\frac{\Delta F_0}{k_B T} - \left(\frac{2}{3}\right)^{1/2} \frac{I^2}{3\pi I_1 I_c} \right] \sinh \left(\frac{I}{2I_1} \right) \quad (1)$$

, where k_B is the Boltzman constant, $I_1 = k_B T / \phi_0$ and ϕ_0 is the superconducting flux quantum. Condensation energy of the miniumum superconducting domain of size $\xi \times \sigma$ is $\Delta F_0 \sim B_c^2 \xi \sigma$, where $B_c(T)$ is the critical magnetic field. The exact expression for the prefactor $\Omega(T, I, H)$ has been defined [1], while being not quantitatively important taking into consideration strong exponential dependence in Eq. (1). Utilizing the above expression for the rate of fluctuations one can get the *time-averaged* effective voltage using the Josephson relation $\langle V \rangle_{PS} = h\Gamma_{PS}/2e$ and the corresponding effective resistance [1]:

$$R_{PS} \equiv \langle V \rangle_{PS} / I = h\Gamma_{PS} / 2eI \quad (2)$$

However, one should keep in mind that the fluctuation-governed resistive state of a 1D superconductor in reality is a dynamic process consisting of discrete events (phase slips) repeated on average with the rate Γ_{PS} . During each phase slip of duration $\sim h/\Delta$ inside the PS core region $\sim \xi$ the magnitude of the order parameter goes to zero and the phase flips by 2π . It is natural to assume that each such fluctuation gives rise to a 'quasi-normal' region where a conversion from non-equilibrium quasiparticles to equilibrium Cooper pairs takes place. The charge imbalance is 'pinned' to each phase slip and decays in time and space. The energy dissipation inside this quasi-normal region should be taken into account to obtain the total effective resistance. The main assumption of the present model is that the processes describing the non-equilibrium state at static normal metal - superconductor interfaces [28] and/or current induced resistive state in 1D superconductors [29] are applicable to the fluctuation-governed 1D superconductivity.

The relaxation of the charge imbalance Q in a 1-D superconducting channel can be described by the differential equation [30]:

$$D\tau_Q \nabla^2 Q = \tau_0 \tau_E \ddot{Q} + (\tau_0 + \tau_E) \dot{Q} + Q \quad (3)$$

, where τ_E is the inelastic electron-phonon collision time, $\tau_0 = 2k_B T_c \hbar / \pi \Delta^2$ is the supercurrent response time, and $D = \frac{1}{3} v_F l$ is the diffusion coefficient, v_F being the Fermi velocity and l the electron mean-free path. In the most general case Eq.(3) describes damped, dispersive waves of charge imbalance. However, for majority of practical applications the low-frequency limit is applicable: $\tau_0 \ll \tau_E \ll 1/\Gamma_{PS}$. In this case the charge imbalance decays on the length $\Lambda_Q = (D\tau_Q)^{1/2}$. The characteristic relaxation time τ_Q is given by [31]:

$$\frac{1}{\tau_Q} = \frac{\pi \Delta}{4k_B T_c} \frac{1}{\tau_E} \left[1 + \frac{2\tau_E}{\tau_S} \right]^{1/2} \quad (4)$$

, where τ_S is the pair-breaking time due to supercurrent or magnetic field. In the most general case the gap parameter is a function of temperature, current and magnetic field $\Delta = \Delta(T, I, H)$. The extra Ohmic contribution associated with dissipation from both sides of the PS core is:

$$R_Q = \rho_Q \left(\frac{2\Lambda_Q}{\sigma} \right) (\tau_0 \Gamma_{PS}) \quad (5)$$

, where ρ_Q is the effective resistivity of the quasi-normal region. In the simplest approach ρ_Q can be considered of the order of the normal state resistivity of the material. The last term in Eq. (5) is the statistical weight accounting for the repetition of PSs with the average rate Γ_{PS} each of duration τ_0 . To obtain the total effective resistance of a 1D channel in the resistive state one should take a sum of Eq. (2) and Eq. (5).

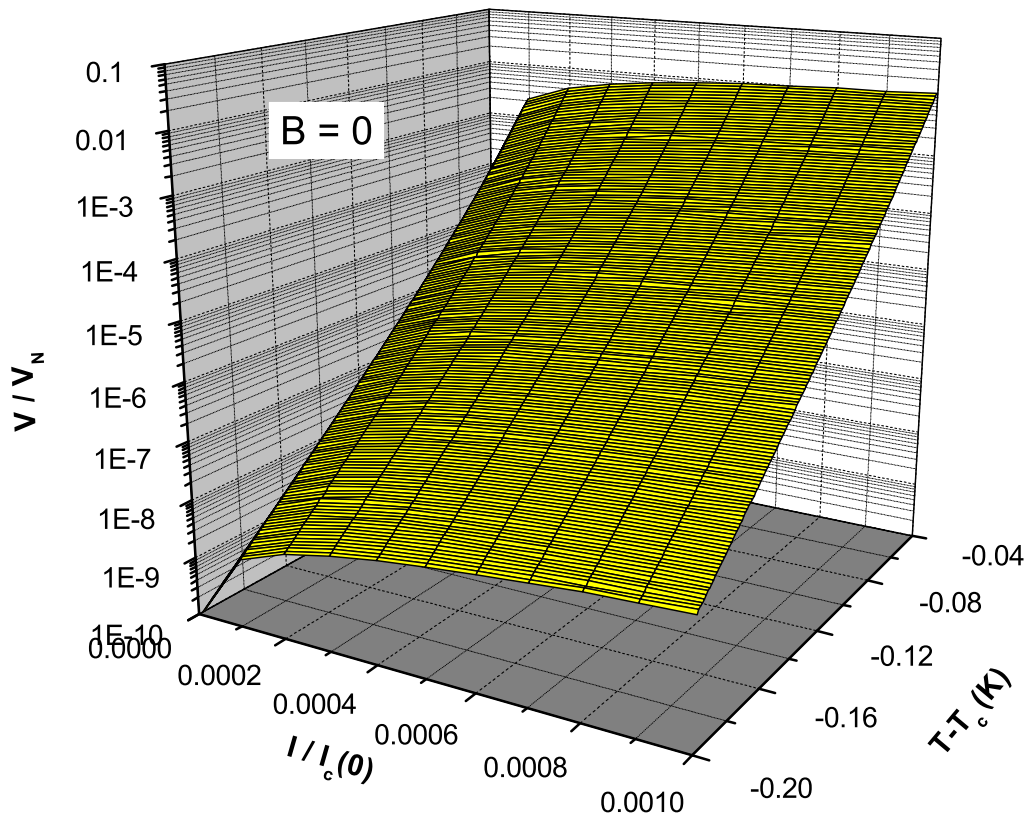


FIG. 1: Effective time-averaged voltage vs. supercurrent I and temperature T in zero magnetic field for $20 \text{ nm} \times 20 \text{ nm} \times 10 \text{ }\mu\text{m}$ aluminum nanowire. Current is normalized by its zero-temperature value $I_c(0)$.

The pair-breaking due to supercurrent [32], [33] is $1/\tau_S^I = D(p_S/\hbar)^2/2$, where p_S is the supercurrent momentum. Using familiar expressions [28] the τ_S^I can be re-written in a more convenient form:

$$\frac{1}{\tau_S^I} = \frac{D}{2} \left[\frac{I}{3\sqrt{3}I_c(T)\xi(T)} \right]^2 \quad (6)$$

As it is clearly seen from Eq. (6) and Eq. (5) the current reduces the quasiparticle relaxation time and, hence, decreases the charge imbalance relaxation length. However, the supercurrent I always brings the rate of thermal fluctuations Γ_{PS} to higher values: Eq. (1). Note that formally one has to consider the variation of the gap $\Delta = \Delta(I)$ with the applied current. In the present paper we neglect this small $\Delta(I)$ deviation which plays no significant quantitative role for calculations. The resulting time-averaged voltage always increases with current (Fig. 1).

The impact of magnetic field on the charge imbalance relaxation is qualitatively similar to supercurrent: magnetic field decreases the pair breaking time [34]:

$$\frac{1}{\tau_S^H} = \frac{1.76k_B T_c}{\hbar} \frac{H^2}{H_c^{\parallel}(0)^2} \quad (7)$$

where $H_c^{\parallel}(0)$ is the zero-temperature parallel critical field. Contrary to the case of pair-breaking currents the variation of the gap with magnetic field $\Delta(T, H) = \Delta(T)[1 - H^2/H_c^{\parallel}(T)^2]^{1/2}$ plays quantitatively significant role. Slightly affecting the Γ_{PS} due to variation of the gap $\Delta(H)$, magnetic field noticeably shifts $\tau_Q(H)$ to shorter values. For each particular set of parameters (material, wire diameter, temperature) there is a range of magnetic fields well below the critical values, where the quasiparticle charge imbalance length (= quasi-normal region) is reduced, while the rate of fluctuations is not affected much. The interplay between these two mechanisms gives rise to the negative magnetoresistance (Fig. 2). Note that the effect should be present in all superconducting systems falling into the 1D limit $\sigma^{1/2} \leq \xi$. However, the experimentally observable finite width of the superconducting transition exponentially depends on the wire cross section σ (Eq. (1)). Hence, observation of the negative magnetoresistance phenomenon requires ultra-narrow superconducting channels, and has been observed so far only in sub-100 nm nanostructures [18], [24], [25].

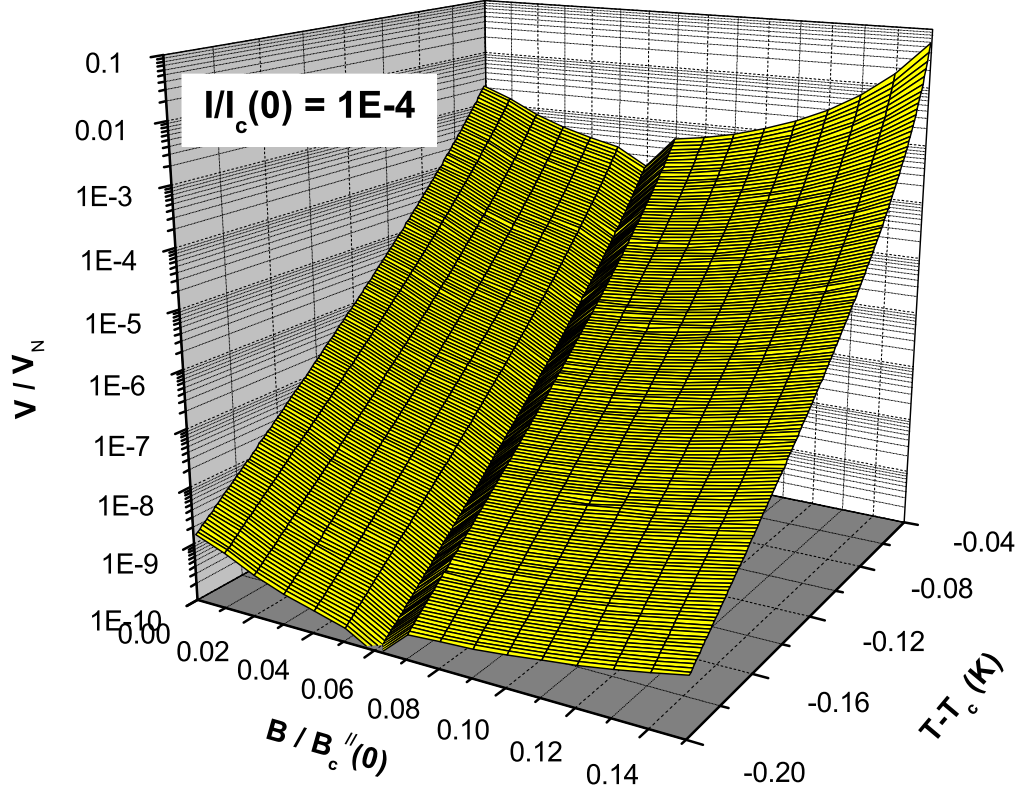


FIG. 2: Effective time-averaged voltage vs. magnetic field B and temperature T at constant DC measuring current I normalized by its zero-temperature value $I_c(0)$ for $20 \text{ nm} \times 20 \text{ nm} \times 10 \text{ }\mu\text{m}$ aluminum nanowire. Magnetic fields are normalized by the zero-temperature parallel critical magnetic field $B_c^{\parallel}(0)$. One can clearly see the negative magnetoresistance developing at small fields.

III. CONCLUSIONS

In conclusion we have presented a simple phenomenological model explaining the negative magnetoresistance observed in ultra-narrow quasi-1D superconducting nanowires in the resistive state. The effect originates from competition of two mechanisms : thermodynamic fluctuations of the order parameter and quasiparticle charge imbalance, which accompanies each phase slip event. First process provides conventional positive magnetoresistance, while the second mechanism gives the negative contribution. The whole concept of thermally ac-

tivated phase slips is applicable sufficiently close to the critical temperature T_c where the utilized formalism of the quasiparticle charge imbalance is valid. Experimentally the nMR effect is observed in superconducting nanowires with very broad superconducting transitions $R(T)$ which can be eventually accounted for a non-thermal PS activation [18], [24], [25]. We believe that the proposed explanation of the nMR phenomenon should be qualitatively valid also in case of the quantum phase slip (QPS) mechanism. However, in this case the extrapolation of the quasiparticle charge imbalance concept down to temperatures well below the critical one requires further justification. It might happen that in this limit the quasiparticle relaxation length Λ_Q should be substituted by the other relevant scale (e.g. coherence length) [35].

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